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REVIEWS

Solitons, Nonlinear Evolution Equations and Inverse Scattering. By M. J. ABLOWITZ and P. A. CLARKSON. Cambridge University Press, 1991. 516 pp. £27.95 or $49.95.

One of the more exciting and fascinating research areas in the last twenty-five years is that of soliton dynamics. Since the initial discovery by Zabusky & Kruskal (*Phys. Rev. Lett.* **15**, 240–243) of some of the amazing properties of solitary waves, through their extensive numerical study of the Korteweg–de Vries equation, and the consequent rediscovery of Scott Russell's pioneering observations of these waves, there has been much effort devoted to exploring the various aspects of those nonlinear wave equations describing soliton dynamics. In general it would seem that the more theoretical aspects of soliton theory have received the most attention. Topics such as complete integrability through the inverse scattering transform, the direct construction of N-soliton solutions, the connection with Bäcklund transformations, the Painlevé test for integrability and the many facets of the deep underlying algebraic structure of these equations, figure prominently in the literature on soliton dynamics. However, it is appropriate to remind readers of the *Journal of Fluid Mechanics* that the soliton equations have often turned out to be of considerable practical relevance as well in a wide variety of physical contexts. The Korteweg–de Vries equation is now well-established as a valid model for the description of weakly nonlinear long waves and the nonlinear Schrödinger equation is equally well-known for the description of small-amplitude wave groups. Together with their respective two-dimensional counterparts, the Kadomtsev–Petviashvili equation and the Davey–Stewartson equation, these equations have been shown to provide valuable insights and useful quantitative information in a wide variety of fluid flow problems. This conjunction of an abstract and deep theory with practical relevance and utility is an aspect of soliton dynamics that has too often been overlooked, although it is an attractive feature of one of the principal early monographs on this topic, namely, *Solitons and the Inverse Scattering Transform*, by M. J. Ablowitz & H. Segur (SIAM Studies in Applied Mathematics, 1981). Indeed, the text under review here is best regarded as a sequel to this, since in particular one of the authors then is also a co-author now.

In the ten years or so since the publication of the monograph by Ablowitz & Segur there have been several major developments in soliton theory. The text under review concentrates primarily on just one of these, namely the inverse scattering transform in two or more spatial dimensions, with particular emphasis on applications to the Kadomtsev–Petviashvili and the Davey–Stewartson equations. Topics of current research interest not considered in depth in this text include the various methods for constructing special solutions, and the important issue of the underlying algebraic structure of the class of integrable nonlinear wave equations. However, these and other omissions are not likely to trouble those readers interested in the application of soliton dynamics to problems in fluid mechanics. Although this text is best regarded as a sequel to the monograph by Ablowitz & Segur, it is self-contained and could be read independently of the earlier work. But, in comparison with that earlier monograph there is perhaps less emphasis on applications here. This reflects the current trend in soliton research where there seems to be a gap between theoretical developments on the one hand and questions of practical application on the other.

This is unfortunate in general, and in particular for fluid problems since at least some of the theoretical developments in the current work on nonlinear wave equations in two or more spatial dimensions are potentially of practical interest. That said, the text under review suffers less from this defect than much of the current literature, and can be recommended to all those readers of this *Journal* interested in nonlinear waves in the context of fluid flows.

The text begins with an introductory chapter which gives an overview of the 'state of the art' for integrable nonlinear equations and their associated inverse scattering transforms. The second chapter describes the inverse scattering transform for the Korteweg–de Vries equation in detail, and touches briefly on other aspects of this equation's integrability. The third chapter describes the general theory for inverse scattering in one spatial dimension. Although a little out-of-place with respect to the general theme, this chapter includes two interesting sections on numerically induced chaos for the nonlinear Schrödinger equation, and on cellular automata. The fourth chapter describes the inverse scattering procedure for the intermediate long-wave equation and the Benjamin–Ono equation. Like the aforementioned Korteweg–de Vries equation and nonlinear Schrödinger equation, these equations are very relevant to the study of nonlinear waves in fluids. In chapter five, one of the longer parts of this text, there is a description of the inverse scattering transform in two spatial dimensions, centred around the Kadomtsev–Petviashvili and the Davey–Stewartson equations. Both these equations are again of great relevance to the study of waves in fluid flows. Chapter six looks at the inverse scattering procedure in several spatial dimensions, and concludes with the tantalizing conjecture that possibly all integrable nonlinear evolution equations are reductions of the self-dual Yang–Mills equations. Chapter seven is rather different from the rest of the text, and discuss the Painlevé test for integrability and some of the properties of the Painlevé equations. The text concludes with a brief discussion of some of the current topics of interest in soliton research. Throughout, this book is well-written and presented.

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Fluid Mechanics. By P. K. KUNDU. Academic Press, 1990. 638 pp. £55.
Viscous Flow. By F. S. SHERMAN. McGraw-Hill, 1990. 746 pp.

Fluid mechanics provides an umbrella under which there is ample room for engineers, mathematicians and scientists. Their collective interest is motivated by a range of goals, from the search for understanding of physical phenomena to the solution of applied problems, and at its frontiers is challenged by the considerable maturity of the subject. How should the future members of this informal society be introduced to fluid mechanics and prepared for the task of continuing progress across the spectrum of possible interests? Given the importance of textbooks and monographs in this process, opinions are offered regularly in this *Journal* as reviewers applaud or deplore particular examples. Such material must, of course, provide the student with certain specific information at a suitable breadth and depth; reasonable people might disagree about choices and details. However, as McNown has argued eloquently in a review in this *Journal* (*J. Fluid Mech.* vol. 15, 1963, p. 154), the more difficult and subtle obligation to prepare the student to approach *new* problems or areas of interest should not be taken lightly. Unfortunately, in spite of good intentions, books often turn out to be merely different ways of meeting the first of these two objectives;

the second is, in effect, left to chance or even ignored. This brings us to the two books in question, which represent quite different (in style and content) examples of what can be accomplished; judged on the basis of their particular contributions, one can be expected to be quite successful while the other will likely have only a modest impact.

The theme of the book by Kundu is 'general fluid mechanics'; it is intended for 'undergraduate and beginning graduate students of science and engineering'. Since no previous knowledge of the subject is assumed, the early chapters necessarily introduce such obvious topics as kinematics and conservation laws. This beginning is followed by additional topics of potential interest to both engineers and geophysicists, including chapters on vorticity dynamics, instability and turbulence. More narrowly focused material intended solely for engineers or geophysicists is also presented. The chapters for engineers, which are essentially interchangeable with those in many undergraduate textbooks, pursue aerodynamics and compressible flow, while gravity waves and geophysical fluid dynamics are provided for geophysicists; items of geophysical interest are sprinkled throughout the book. There is a total of 15 chapters, which vary in length from less than 20 pages (dynamic similarity) to about 70 pages (geophysical fluid dynamics), and three short appendices; somewhat over 100 exercises are provided.

The organization of this broad range of topics into a single volume has not left a lot of room for depth or originality. Much of the book is yet another largely routine compilation of material that is already available in a number of competitors, and there can be little disagreement with the author's characterization of portions of it as being 'standard'. The experienced teacher will find few surprises in the well-worn paths over which this 'standard' material travels while, for the uninitiated student, the same journey will often seem mysterious and incomplete. Although there is merit in some of the more advanced material of the later chapters, it rests on a foundation that is inadequate for either careful understanding or independent progress.

A few specific criticisms, samples from a longer list, provide a sense of the clarity and depth of the 'standard' presentation. The short introductory chapter is similar in outline to that in many undergraduate textbooks, but buoyancy forces are not mentioned in spite of their implicit use in a discussion of static equilibrium (p. 18); this is a surprise since the book seems to lean, at least in part, toward the interests of geophysical fluid dynamics. The unqualified statement (p. 9) that 'It is well-known that the free surface of a liquid in a narrow tube rises above the surrounding level due to the influence of surface tension' ignores the example of a mercury manometer. In the chapter on kinematics, as part of a terse, two-page treatment of the concepts of Eulerian and Lagrangian descriptions and material derivative, we find (p. 49) 'Employing Eulerian coordinates (x, y, z, t), we seek to calculate the rate of change of F at each point following a particle of fixed identity'. Unfortunately, in spite of its significance, no motivation for such a calculation is provided. In the same chapter, the expression for fluid acceleration is oddly placed in an exercise, and one wonders what a student will make of 'The word flux is generally used to mean the surface integral of a variable' (p. 57). More careful writing would have avoided the conflict between 'Irrotational flow around a finite three-dimensional object is unique because there is no circulation' (p. 168) and the later treatment (p. 564) of lifting-line theory for wings of finite span which naturally involves a 'circulation distribution'. Finally, the streamwise dependence of the velocity components in a boundary layer is incorrectly ignored in writing them as $u(y)$ and $v(y)$ (pp. 317–318). In general, the book also suffers from a large number of errors that should have been noticed during the production process; many of them are minor, but their total does not inspire confidence.

In contrast to the routine nature of the 'standard' material, the chapters on gravity waves, instability, turbulence, geophysical fluid dynamics and part of the chapter on boundary layers and related topics form a more satisfying and somewhat unusual combination that shows greater attention to motivation and detail. Various developments are more careful and complete, and one senses that the study of fluid mechanics might be interesting and rewarding. In particular, references to the periodical literature are introduced, as are a few photographs, and, rather than unsupported phrases like 'experiments show', specific examples of experimental evidence appear more regularly. It is unfortunate that (at least) the same level of care was not used in the other portion of the book since, as it stands, it is as if two books were written by two different authors and then merged. The marriage is not a happy one and the book seems best suited to a special audience that has enough background in the subject so that the early chapters can be passed over.

Sherman has the advantage of pursuing a narrower topic, 'the first year of study of viscous flow', an important and central part of fluid mechanics. His book is written for 'seniors and graduate students who had adopted fluid mechanics of heat transfer as major fields of study', and have completed an introductory course in the subject. The book naturally divides into two parts: (i) the first eight preliminary chapters (about 125 pages); and (ii) the remaining six chapters, which are titled analytical solutions of the full Navier–Stokes equations, numerical solutions of the full Navier–Stokes equations, creeping flows, laminar boundary layers, instability of viscous flows, and turbulent flow. There are two appendices, one on mathematical aids and a much longer one which contains 16 FORTRAN programs for use in various problems introduced in the book. Nearly 200 exercises of varying difficulty and with various goals are provided. Many of the chapters conclude with useful 'sample calculations', which serve to amplify certain topics and/or treat specific examples. The well-known instructional motion pictures on fluid mechanics are recommended in appropriate places and extensive references to the periodical literature are provided.

Because of the expected prior knowledge, the short chapters of part (i) are very concise and may prove to be difficult for those who do not have solid backgrounds. Examples of their titles include kinematics of rotating and deforming flow, conservation equations, Newtonian fluids and the Navier–Stokes equations, and flows with nearly constant density and transport properties. They serve to enhance and generalize the reader's existing knowledge of fluid mechanics, to establish some common language and the desired mathematical level, and, perhaps most importantly, to start to orient thinking in the specific directions that the author intends to emphasize in the more detailed chapters of part (ii). The latter goal is particularly obvious in the chapter on vorticity, the longest of the early chapters at about 30 pages, since vorticity plays an important recurring role. This chapter includes a brief discussion of the random-vortex method for the numerical simulation of two-dimensional viscous flows. While some might question its relevance or utility, it is only one example of the many new approaches and fresh insights that the author has carefully integrated in the book.

The final six chapters form the heart of the book; their titles include several old favourites. However, since the author has apparently thought deeply about viscous flow, this is not just another version of the usual time-worn manipulations and results. Rather he has produced a carefully planned and organized book that contains original and timely points of view in many places. The non-traditional emphases and interesting writing should stimulate even experienced readers, and students will be rewarded for the time spent thinking about the book's contents. The author is confident enough to identify areas where understanding is limited and further work is needed.

Instead of being presented as an assortment of unrelated examples, the analytical solutions of the Navier–Stokes equations in chapter 9 have been organized around the mechanisms of diffusion and convection. The chapter on numerical solutions will not produce specialists in computational fluid dynamics, but, in the typical manner of the book, provides a useful sense of the promise and difficulty of such activity. In addition to the expected topics, e.g. lubrication theory and Stokes flow, there is also a section on 'slender viscous films with free surfaces' in the next chapter on low-Reynolds-number flows. The two main predictive tools in the chapter on laminar boundary layers, first Thwaites' method and then numerical methods, help to give a new twist to this classical concept. This use of numerical methods is fairly extensive, and lessons about their limitations continue. The chapter ends with discussions of unsteadiness, three-dimensionality, separation, and higher-order theory. In the next chapter, the treatment of instability begins by linking it to vorticity. Detailed studies of parallel-flow and centrifugal instability are next, and there is a brief section of 'viscous fingering'. The chapter closes by considering the connection between instability and transition. The final chapter is devoted to turbulent motion. After some introductory remarks, various 'cartoons of turbulence' are used to motivate some appreciation of this motion; again vorticity is important. Various empirical results, e.g. the law of the wall, associated with wall and free turbulence follow, but it will be necessary to consult more specialized sources for information on current practice in turbulence modelling. A descriptive discussion of the numerical simulation of turbulent flow is the final major topic.

All in all, the author has indeed shown that 'viscous flow is my personal favorite'. His *Viscous Flow* is a book to be read and studied in a quiet place with a pad and pencil close at hand. There is no risk in saying that it will be difficult to teach from parts of it; those who recall the review by McNown will appreciate the compliment.

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